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# ALGORITHM FOR SPACECRAFT ANGULAR AND TRANSLATIONAL MOTION CONTROL WITH USE OF ORIENTATION THRUSTERS

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**Abstract**

The paper discusses the algorithm of spacecraft orientation and docking thrusters control for simultaneous spatial and angular motion. The solution of control velocity formation problem and the problem of required engines configuration determination along with the optimization of control vector execution accuracy are considered. The formation of control velocity is carried out using a phase plane with switching lines and a zone of inactivity. The calculation of thrusters working duration time is based on the method of least squares with non-negative resulting solution vector and additional boundary conditions. In the paper, the necessary control parameters were chosen to ensure the necessary accuracy of spacecraft stabilization. To demonstrate the developed algorithm, mathematical modelling of various considered spacecraft's orbital flight stages was executed, including damping of initial angular velocities, spatial motion, and stabilization under the influence of continuous perturbations. The simulation took into account the disturbing moments acting on the spacecraft, thrusters mounting errors and the characteristics of the angular velocity meter. The elastic characteristics of the structure were not taken into account. The results of mathematical modelling showed that the proposed algorithm coped well with the task, and was able to ensure the movement of the spacecraft center of masses in a given direction and simultaneous angular stabilization with required accuracy

**Keywords**

*Orientation thrusters, simultaneous control of angular and spatial motion, least squares method, control velocity formation, calculation of thrusters burn duration*

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**Introduction.** Let us consider an algorithm for controlling the motion of a spacecraft using a propulsion system, whose tasks also include docking with the orbital station. The algorithm should provide simultaneous control of the ship mass center translational and rotational motion. Suppose that the propulsion system consists of 30 docking and orientation thrusters, with a thrust of ~ 245 N (used to control the spatial and rotational movement of the mass center); and reboost thrusters of greater thrust (to control only mass center spatial movement). It is assumed that to ensure redundancy, all docking and orientation thrusters are divided into two full-size collectors, each of which is sufficient to control the movement in all six degrees of freedom, but the simultaneous use of all 30 docking and orientation thrusters is considered as a standard configuration. The control algorithm described in this paper does not consider the use of reboost thrusters.

The task of controlling the motion of a spacecraft using thrusters consists of two main parts [1]: determining the required change in speed at each control clock of the onboard central computer and implementing the required change in angular velocity using the propulsion system by choosing the optimal engine operational set. The principle of solving each problem will be described further.

**The law of the control velocity formation.** The principle of solving the problem of the control speed formation for controlling the motion around the center of masses can be illustrated using the phase plane [1]. For each control channel on the phase plane, the control law is formed. Let the misalignment angle $i = x,\ y,\ z$ between the given and the real position of the spacecraft be plotted on this plane on the $X$-axis $\nu_i$, and the ordinate is the difference between the current estimation of the angular velocity projection on the control axes and the angular velocity of the programmed motion (for example, it can be the velocity of the orbital motion) $\omega_{bi}$, $i = x,\ y,\ z$. The estimation of the angular velocity $\hat{\omega}_i(t)$ was obtained using the indications of the angular velocity sensor $\omega_{meas}(t)$, $i = x,\ y,\ z$ by means of the elastic oscillation filter, known as Lyuinberger filter [2–4]. To reduce fuel consumption, a dead band was used in the control law, within which there was no control.

The switching lines in the coordinates $\nu_i$ and $\hat{\omega}_i(t) - \omega_{bi}$ limiting the dead zone (white area), whose width along the vertical line is $\delta\omega = \pm\ 0.12\ ^\circ/\mathrm{s}$, are shown in Fig. 1. The angle deadband can be set by parallel shift of the switching lines horizontally to both sides of the ordinate axis.

Further, without loss of generality, we will assume that the control channels are located along the angular velocity sensor sensitivity axes. The control law will

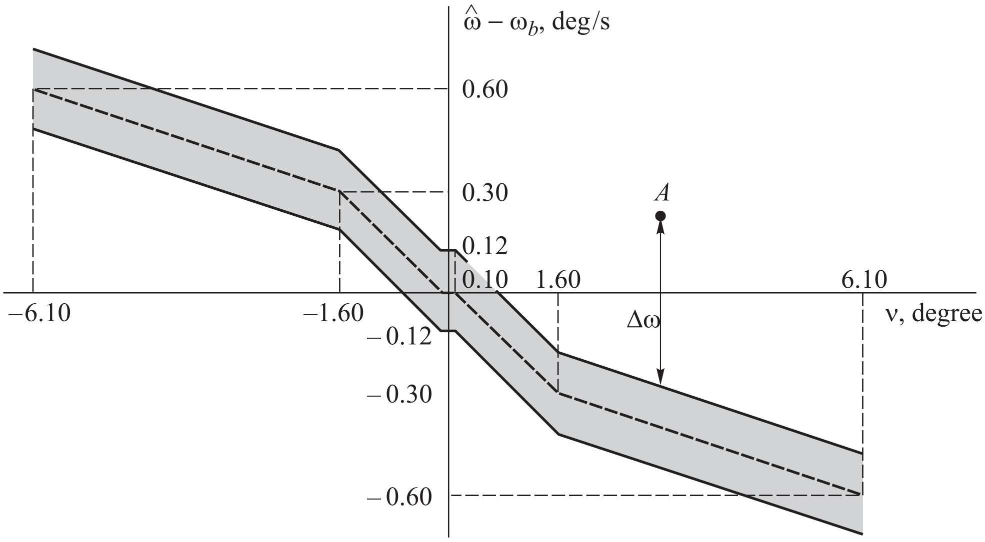



**Fig. 1.** Switching lines on the phase plane

be chosen based on the magnitude of the mismatch angle relative to the required orientation, as it is done in [1, 5]. As a result, at small misalignment angles, for each control channel the required increment of the angular velocity (control signal $u_i = \Delta\omega_i$, $i = x, y, z$) is defined as the vertical distance from point $A$, corresponding to the current phase state of the system, to the nearest border of the dead band, taking into account the hysteresis:

$$u_i = \Delta\omega_i = \hat{\omega}_i - \omega_{bi} - \frac{\delta\omega_{cl}}{k_a}, \; i = x, y, z, \tag{1}$$

where $k_a \geq 1$ is the coefficient of angular hysteresis; $\delta\omega_{cl}$ is the nearest boundary of the dead zone in angular velocity.

In case of large angular mismatches between the given and current orientation, exceeding 15° in total over all control channels, a turn is formed around Euler axis [6]. In this case, the control signal is determined by the formula

$$u_i = \Delta\omega_i = \hat{\omega}_i - \omega_{bi} - \omega_0 \frac{N_i}{\sqrt{1 - N_0^2}} - \frac{\delta\omega_{cl}}{k_a}, \; i = x, y, z, \tag{2}$$

where $\omega_0$ is the angular velocity of the turn; $\mathbf{N} = \left(N_0, N_x, N_y, N_z\right)^t$ is the current angle mismatch quaternion. To minimize fuel consumption in the process of overshoot, new readjusted law of control [1, 5] is introduced as the angle approaches the dead zone, which is shown in Fig. 1. In this case, if the angular mismatch $\nu_i$ in the channel $i = x, y, z$ exceeds 6.1° and the total angular mismatch of all channels is no more than 15°, then the switch line in this control channel is determined by the motion with constant acceleration $e_i$. From the condition of changing the angle when moving with constant acceleration

$$\nu_i = e_i t^2 / 2,\ i = x,\ y,\ z,$$

time can be expressed and, by substituting it into the law of change in angular velocity, the switch line is formulated

$$\omega_i = \sqrt{2 e_i \nu_i},\ i = x,\ y,\ z.$$

Thus, in this case the control signal will be determined according to the formula

$$\Delta\omega_i = \hat{\omega}_i - \omega_{bi} - \operatorname{sign}(\nu_i)\sqrt{2 e_i \nu_i} - \frac{\delta\omega_{cl}}{k_a}. \quad (3)$$

As a result, we obtain the vector of the required increments of the angular velocity

$$\Delta\boldsymbol{\omega} = \left[\Delta\omega_x,\ \Delta\omega_y,\ \Delta\omega_z\right]^t.$$

While controlling the angular motion of a spacecraft, along with the control moments, secondary forces are also created that affect the vehicle orbit; the orbital disturbances compensation mode was introduced to reduce this perturbation. While controlling the motion of the spacecraft mass center in this mode, the linear accelerations created by the thrusters are continuously integrated. While the integral increment of speed stays in the interval [–1 cm/s, 1 cm/s], the control action is not formed, and when this increment exceeds the allowed interval boundaries, the control speed is formed by the formula

$$\Delta V_i = \int_{t_0}^{t} a(t)\,dt - \frac{\delta V_{cl}}{k_l}, \quad (4)$$

where $a(t)$ is the linear acceleration created by all thrusters at the current clock of the onboard central computer; $t_0$ is the start time of orbital disturbances compensation mode; $\delta V_{cl}$ is the nearest limit of the linear velocity dead zone; $k_l \geq 1$ is the linear hysteresis coefficient. At the absence of orbital disturbances compensation (4), the control velocity is defined from outside and calculated in the algorithms of approach or center of masses motion control.

**The choice of the optimal thrusters engaging configuration.** After determining the required change in speed of the spacecraft at each control clock of the on-board central computer, it is necessary to execute this change using the propulsion system by choosing the optimal engine engaging configuration. The required values of linear and angular velocity components increments and accelerations created by each engine in each control channel will be the initial data for the algorithm. The control is formed simultaneously for six control channels of the mass center translational and angular motion. To calculate the control accelerations, we introduce the following notation: $n$ = 30 is the total number of thrusters available, $r_i = \left[ r_x^i, \; r_y^i, \; r_z^i \right]^t, \; i = 1, \ldots, n,$ is the mounting coordinates of the $i$-th thruster in the coordinate system associated with the spacecraft; $r_{cm} = \left[ r_x^{cm}, \; r_y^{cm}, \; r_z^{cm} \right]^t$ is the position of the spacecraft center of masses in the same coordinate frame; $F_i = \left[ F_x^i, \; F_y^i, \; F_z^i \right]^t, \; i = 1, \ldots, n,$ is the vector of thrust generated by the $i$-th docking and orientation thruster; $M_i = \left( r_i - r_{cm} \right) F_i = \left[ M_x^i, \; M_y^i, \; M_z^i \right]^t, \; i = 1, \ldots, n,$ is the moment created by the $i$-th thruster relatively to the axes of the coordinate system $OXYZ$ associated with the spacecraft. At the coordinate format, the moment created by thrusters can be represented as follows:

$$
\begin{gathered}
M_x^i = \left( r_y^i - r_y^{cm} \right) F_z^i - \left( r_z^i - r_z^{cm} \right) F_y^i, \\
M_y^i = \left( r_z^i - r_z^{cm} \right) F_x^i - \left( r_x^i - r_x^{cm} \right) F_z^i, \\
M_z^i = \left( r_x^i - r_x^{cm} \right) F_y^i - \left( r_y^i - r_y^{cm} \right) F_x^i, \\
i = 1, \ldots, n.
\end{gathered}
$$

The values of angular $\varepsilon_i = \left[ \varepsilon_x^i, \; \varepsilon_y^i, \; \varepsilon_z^i \right]^t$ and linear $a_i = \left[ a_x^i, \; a_y^i, \; a_z^i \right]^t$ accelerations are calculated directly using

$$
\begin{gathered}
a_i = F_i / m, \; i = 1, \ldots, n; \\
\varepsilon_i = J^{-1} M_i, \; i = 1, \ldots, n.
\end{gathered}
$$

Here $m$ is spacecraft's mass; $J$ is its inertia tensor. As a result, from linear $a = \left[ a_1, \; a_2, \; \ldots, \; a_n \right]$ and angular $\varepsilon = \left[ \varepsilon_1, \; \varepsilon_2, \; \ldots, \; \varepsilon_n \right]$ accelerations created by each thruster in projections onto the associated coordinate system, we obtain the matrix of size $6 \times n$:

$$
\mathbf{A} = \begin{bmatrix} a \\ \varepsilon \end{bmatrix}.
$$

The matrix **A** contains the coefficients of the thrusters influence on all control channels.

Let's consider the following system:

$$
\mathbf{A} t = b, \tag{5}
$$

where $t = \left[ t_1, \quad t_2, \quad ..., \quad t_n \right]^t$ is vector of docking thrusters burn durations, and $b = \left[ \Delta V^t, \quad \Delta\omega^t \right]^t$ is vector of required increments of linear and angular velocities. As a result, to achieve the goal of control, it is necessary to form such a vector *t*, which would provide the specified increments of speeds *b*.

This problem can be solved in several ways (catalog method, linear programming [1]). In this paper, the least-squares solution is proposed [7, 8] with variable constraints implemented in the algorithm Bounded Variables Least Squares (BVLS) [9, 10].

The BVLS algorithm was developed in the 1980s by American engineers Stark and Parker and was based on the Non-Negative Least Squares (NNLS) algorithm [11]. A distinctive feature of the BVLS algorithm is the ability to impose any numerical restrictions on the components of the solution vector, which is basically required in this case, since the task of determining the thrusters burn duration is solved every computational clock of the onboard digital machine, which limits the maximum time of the burn with the duration of this clock. Let's reformulate problem (5) to apply the BVLS method. As the minimized functional, we take the norm of the divergence vector between the algorithm output result and the input vector *b*:

$$J = \left| \mathbf{A}t - b \right|^2 \to \min. \tag{6}$$

We impose the following restrictions on the components of vector *t*:

$$ub \geq t_i \geq lb, \tag{7}$$

where $ub$ and $lb$ are restrictions, respectively, from above and below, imposed on the components of the solution vector. In this case, $lb = 0$ and $ub = 1$. The restriction $t_i = lb = 0$ means that the $i$-th engine is not turned on at a given clock, $t_i = ub = 1$ means that the $i$-th engine is working during the entire control clock. BVLS algorithm and its block diagram are shown in [9].

**The results of mathematical modelling.** To estimate the applicability and test the functionality of the proposed control algorithm, mathematical modeling of the spacecraft motion was carried out. The simulation was performed with the approach used in the development of real spacecraft on-board software [12, 13]. A flowchart of the modeling process is shown in Fig. 2.

The modelling complex includes following models: the dynamics of linear and angular motion; angular velocity meter and propulsion system; redundant orientation control loop, which was used to create disturbing forces and moments. In the "Algorithm of Control" block, the required increments of angular and linear velocities of the spacecraft were calculated using (1), (4), and in the "Algorithm of docking and orientation engines engaging" block, the docking

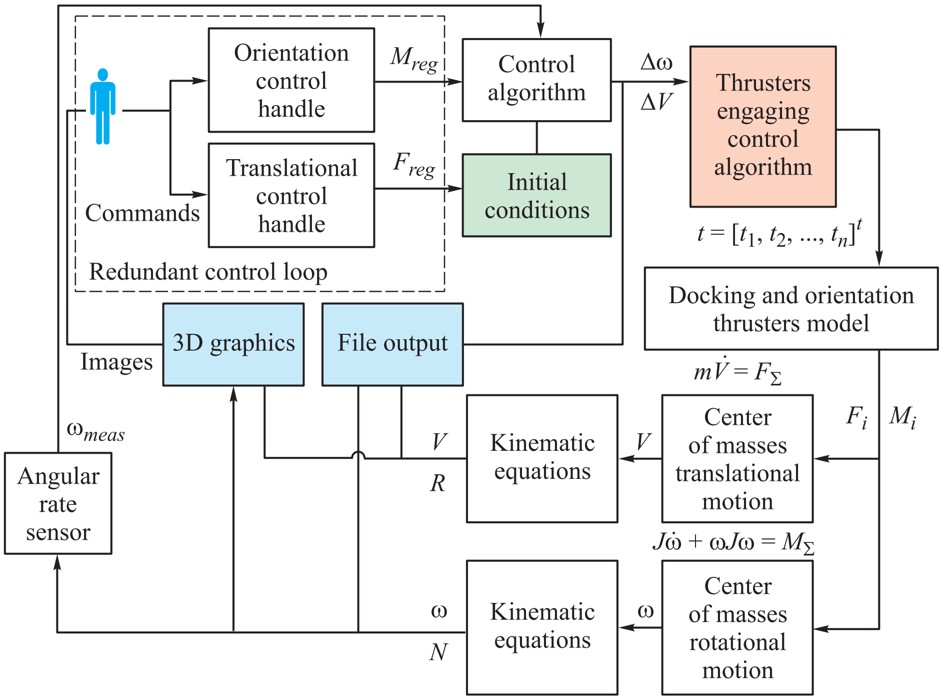



**Fig. 2.** Model of motion control loop

thrusters burn duration is directly calculated by minimizing the functional (6) with constraints (7) using the BVLS algorithm. To display the results of the simulation and simulation process control, service software was used. The simulation took into account the effect of gravitational and aerodynamic forces and moments, but didn't consider the elasticity of the structure.

The simulation was performed according to the thrusters' layout, similar to that given in [14, 15]. A full set of 30 thrusters was used for control. Pulse profiles, measurement and command execution delays, mounting and thrust errors were simulated according to [1].

So, the mathematical simulation of docking and orientation thrusters engaging control algorithm was performed in two flight modes. As the first mode the orientation building and maintaining were simulated — non-zero initial velocities damping and the initial angular mismatches elimination in order to build the orbital coordinate system [16], with further maintaining of such orientation. As the second flight mode, the stabilization mode was simulated under the conditions of continuous perturbation — for example, when thrusters had been working to move a spacecraft, creating side effects in the orientation channels. The results of the algorithm operation were given in the form of a phase portrait — the dependence of the signal on its derivative, in this case, the angle on the angular velocity.

Fig. 3 shows the algorithm simulation results in the mode of eliminating the initial angular mismatches (building orientation) and zero deviations maintaining for 1 turn (~ 90 min).

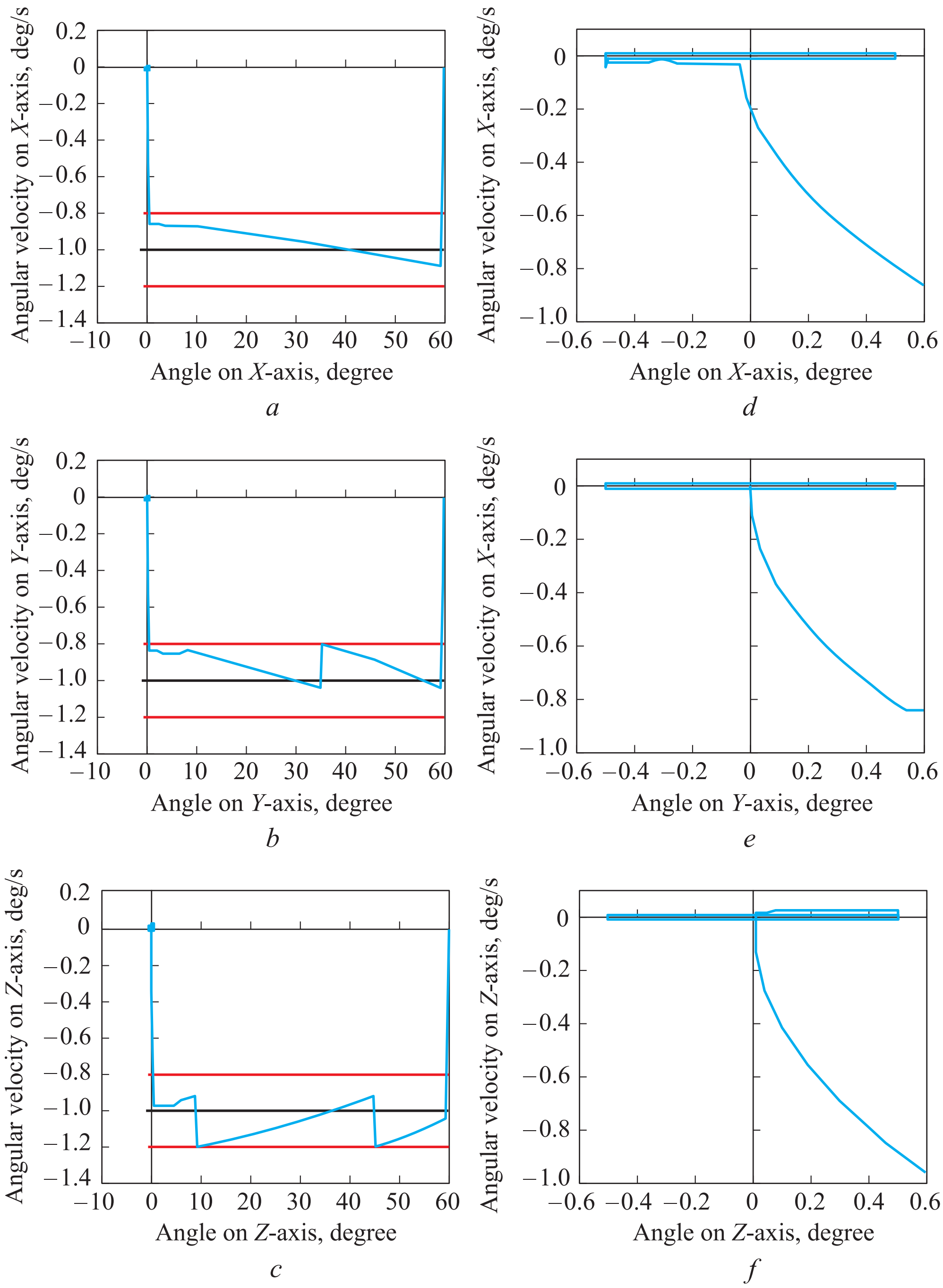
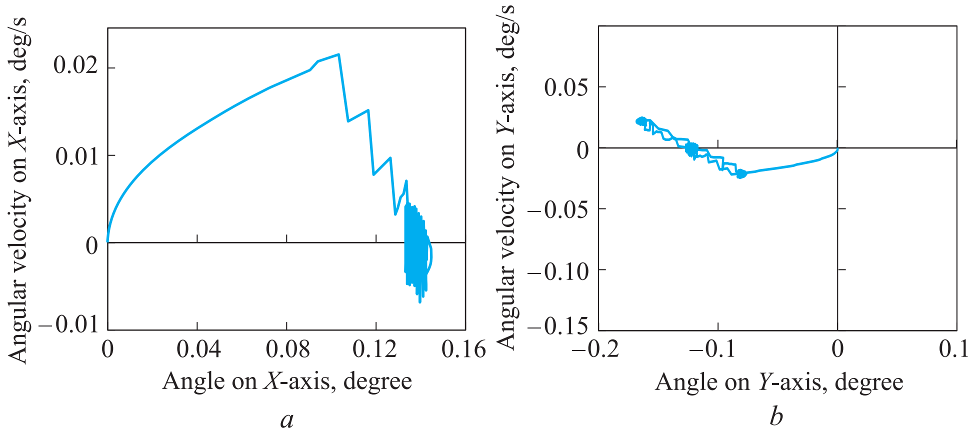



**Fig. 3.** Modelling results for orientation building mode (*a*, *b*, *c*) and orientation maintaining mode (*d*, *e*, *f*)

Fig. 4 shows the simulation results in the mode of continuous disturbance compensation. This mode is used, for example, during manual motion control using the backup motion control loop. Disturbances in the orientation channels appear from the thrusters operation, which must be immediately compensated, since the image that the operator uses to control the spacecraft in manual mode will immediately react to perturbations in the orientation channels. For this reason, during manual control, strict requirements are imposed on the orientation perturbations compensation mode — the angular mismatch must not get beyond the boundaries of 0.1–0.2 degrees, and the angular velocity must be maintained within 0.1 deg/s. Higher angular velocity will be determined by operator as the “trembling” of image, which makes it difficult or even impossible to control.



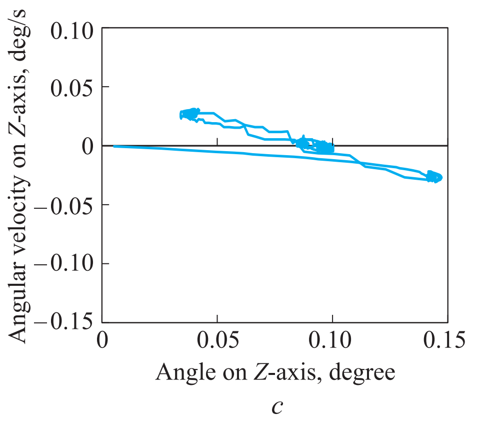




**Fig. 4.** Modelling results for the continuous perturbations compensation mode

The simulation results (see Fig. 3, 4) show that the requirements posted above were met. Orientation was successfully built and maintained, the angular velocity stayed within acceptable limits. The fuel consumption when using the BVLS algorithm was comparable to the fuel consumption when applying catalog method to this thrusters layout (within 10–20 %).

**Conclusion.** As part of this study, we tested the possibility of using an algorithm based on the least-squares method with variable constraints to control the engaging of docking and orientation thrusters to simultaneously control spacecraft translational and angular motion. Using the developed software component that simulates the external environment and the onboard equipment, the modes of building and maintaining orientation were tested, as well as the mode of compensating the action of continuous disturbance in the orientation channels during motion manual control. Based on the obtained phase portraits, conclusions were made about the possibility of using the algorithm for controlling the spacecraft, and the requirements for the manual control stabilization algorithms were confirmed. The least-squares algorithm is a promising way to solve the problem of thrusters engaging configuration optimal choice with great potential for further research and improvements such as the implementation of dynamic control.

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