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SIGNAL EMITTERS LOCALIZATION BY SPECTRAL METHODS

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Abstract

Methods are considered of generating signals in a radar sensor receiver provided with a colocated antenna system of *Multiple Input Multiple Output (MIMO)* configuration when synthesizing virtual antenna arrays. Review of the basic spectral methods used in beamforming with digital automobile radars, such as Capon method, *MUSIC* and *ESPRIT* was carried out. Spatial spectra obtained by various methods were constructed for a radar signal model, and comparative spectra characteristic by angular resolution is provided. Output heatmaps in the angle-distance coordinate were constructed to represent methods operation according to the real target situation based on data sampling from the front-mounted automobile radar. Advantages of decomposition methods based on *MUSIC* and *ESPRIT* algorithms in solving the problem of a signal emitter localization are presented

Keywords

Beamforming, localization, signal, virtual antenna array, spectral evaluation, MIMO

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Introduction. Development of the Advanced Driver Assistance System (ADAS) indicates increased requirements to its components. In order to provide the necessary functions, such as active cruise control (ACC), autonomous emergency braking system (AEBS), etc. in various road conditions, the radar sensor used in ADAS should be provided with acceptable resolution to determine not only the distance to the located object and its speed, but also the angular position. This is necessary to assess position of oncoming and passing vehicles in direct and indirect sections of a road, selection of bump stops and transparencies, etc.

In recent years, developers of automobile radars are building systems with several transmitting and receiving channels in order to increase the angular resolution of radiation emitter localization (determining direction-of-arrival (DOA)). In this case, the transceiver module (TM) is presented in *Single Input*

Multiple Output (SIMO) or *Multiple Input Multiple Output (MIMO)* configuration. The modern element base makes it possible to implement the *MIMO* 2Tx/4Rx or the 2Tx/4Rx configuration on the basis of a single integrated front-end module; at the same time, cascaded solutions based on several TMs are encountered to obtain super resolution in azimuth and elevation. Such spectral evaluation methods as *MVDR, MUSIC, ROOT-MUSIC, ESPRIT, TLS ESPRIT*, etc. are used in the process of digital signal processing for target localization. Methods most commonly used out of these are described below in the context of digital beamforming in automobile radar sensors; besides, their comparative characteristics are presented based on model and sampling data obtained from the real target environment.

Signal model. Methods of creating a virtual phased antenna array are described in detail in works [1–3]. To simplify, let us consider a linear antenna array with uniform elements distribution consisting of M transmitting and N receiving elements oriented along the x axis (Fig. 1). Let us assume that in small antenna arrays directions of the signal arrival are the same for each element and are characterized by the Θ angle [4].

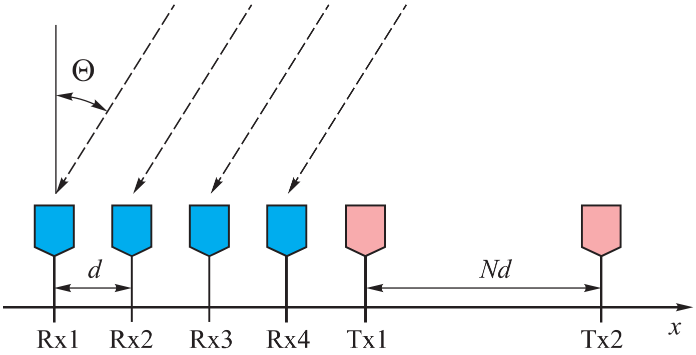

Fig. 1. Radar antenna array configuration diagram

Signal is received by the first sensor at the origin of coordinates:

$$S_0(t) = A\exp(j2\pi f_s t + \phi) + n_0(t), \tag{1}$$

where A is the amplitude; $f_s = c/\lambda$ is the narrowband signal frequency and ϕ is the phase at the origin of coordinates at the $t = 0$ time moment.

Signals received by the remaining sensors are connected to the signal (1) received at the origin of coordinates by a phase shift, which depends on the sensor location and for a uniformly distributed linear antenna array has the following form [5]:

$$S_m(t) = S_0(t)e^{-j\cdot 2\pi(m-1)d\sin\Theta/\lambda} + n_m(t).$$

Here $m = 1, \ldots, M$ is the number of the antenna array receiving element and d is the distance between the adjacent elements.

Signal received by a single sensor from several directions:

$$x_m(t) = \sum_{k=1}^{D} S_k(t)\, e^{-j \cdot 2\pi (m-1) d \sin \Theta_k / \lambda} + n_m(t) = \sum_{k=1}^{D} a_m(\Theta_k) S_k(t) + n_m(t), \qquad (2)$$

where $k = 1, \ldots, D$ is the index of direction to the emitter source; $n_m(t)$ is the noise and $a_m(\Theta_k)$ is phase incursion for the m element of the k-th direction.

Signal from the entire receiving antenna array in the vector-matrix form:

$$X = AS + N. \qquad (3)$$

Here $X = [x_1(t),\ x_2(t), \ldots,\ x_M(t)]^T$ is the observation vector — the output signal from all receiving elements of the antenna array at the t time;

$$A = [a(\Theta_1), a(\Theta_2), \ldots, a(\Theta_D)] =$$

$$= \begin{bmatrix} 1 & 1 & \ldots & 1 \\ e^{-j\varphi_1} & e^{-j\varphi_2} & \ldots & e^{-j\varphi_D} \\ \ldots & \ldots & \ldots & \ldots \\ e^{-j(M-1)\varphi_1} & e^{-j(M-1)\varphi_2} & \ldots & e^{-j(M-1)\varphi_D} \end{bmatrix}$$

is the steering matrix, $\varphi_k = 2\pi d \sin \Theta_k / \lambda$; $S = [S_1(t), S_2(t), \ldots, S_D(t)]^T$ are the signals in each direction and $N = [n_1(t), n_2(t), \ldots, n_M(t)]^T$ is the noise.

The task of estimating the signal arrival direction from the object under study is reduced to finding the desired Θ_k angle by the criterion of the X received signal maximum power. Obviously, precision of the estimate and resolution in the angle would depend on the number of receiving channels. An increase in resolution is possible due to the use of MIMO configuration and of such a concept as a virtual antenna array [1–3, 6]. Having a transceiver with M receiving antennas (subarrays), which phase centers are spaced apart from each other at the $d = \lambda / 2$, distance, and with the N transmitting antennas (subarrays) spaced at the $M\lambda / 2$, step, a virtual antenna array is synthesized equivalent to a system with the MN receiving channels.

Transmitter uses time division multiplexing of channels, and the object under study is exposed sequentially to each element of the transmitting antenna system. In turn, each exposure cycle is accompanied by synchronous signal sampling in all receiving channels and by data accumulation in the memory over several periods of the probing signal with linear frequency modulated continuous wave (FMCW) chirps generating a three-dimensional receiver — distance – time data set. After Doppler processing, a data set is generated for each receiving channel in the distance – speed coordinates [2, 3]. In case a signal is registered from a moving source in a scheme with time division multiplexing of transmitting channels, additional phase compensation is required due to the Doppler effect [2].

Direction estimation. The simplest method in estimating direction of the signal arrival is the discrete fast Fourier transform (FFT) from the amplitude-phase array distribution (2) with solution in regard to Θ [3]:

$$P_k = \sum_{k=0}^{D-1} x_m e^{-j\cdot 2\pi(m-1)d\sin\Theta_k/\lambda}.$$

The desired angle is equal to:

$$\Theta = \arcsin(k_{\max}\lambda / d),$$

where $k_{\max} = \max\{P_k\}$.

This method of estimation, despite a probability of efficient implementation from the point of view of computational cost, possesses low resolution capability. In this regard, parametric methods of spectral estimation are used, i.e., minimum dispersion (Capon method, or *MVDR*), *MUSIC* and *ESPRIT*.

Minimum dispersion (Capon method). For the first time, method of spectral estimation by the minimum dispersion (MD) criterion was proposed by D. Capon for processing the spatial temporal signals of seismic sensors [7]. Currently, the method is widely used in tasks of signal emitters' localization (DOA estimation).

MD estimation of the spatial spectrum is written down as follows [8]:

$$P(\Theta) = \frac{1}{a^H(\Theta) R_x^{-1} a(\Theta)}, \tag{4}$$

where $a(\Theta)$ is the steering vector and $R_x = E[XX^H]$ is the correlation signal matrix.

As a rule, the R_x matrix could not be practically obtained directly and is replaced by a correlation matrix resulting from the input implementation vectors (observation vectors) [8]:

$$\hat{R}_x = \frac{1}{N}\sum_{i=1}^{N} x(i) x^H(i). \tag{5}$$

Here N is the number of measurements, which, as a rule, is limited by the number of channels in speed (number of the probing FMCW signal accumulated chirps).

***MUSIC* method (*Multiple Signal Classification*).** This method belongs to decomposition methods of spectral estimation and possesses high frequency resolution. The method main idea lies in separating the observation space into two orthogonal subspaces, i.e., signal and noise subspaces. Estimation algorithm for the signal arrival direction based on *MUSIC* and its modifications were considered in works [9–11].

Assuming that signal and noise are not correlating, and the noise is the white Gaussian, correlation matrix of the observation vector (3) could be written down in the following form:

$$R_x = E\left[(AS+N)(AS+N)^H\right] = AR_sA^H + R_N = AR_sA^H + \sigma^2 I, \qquad (6)$$

where σ^2 is the noise dispersion and $I = \mathrm{diag}(1,1,...,1)$.

In this case, matrix decomposition (6) into eigenvalues and eigenvectors [12]:

$$R_x = AR_sA^H + \sigma^2 I = U_s\Lambda_s U_s^H + \sigma^2 U_n {U_n}^H.$$

Here U_s, U_n are the eigenvectors of signal and noise subspaces and $\Lambda_s = \mathrm{diag}\left(\bar{\lambda}_1, \bar{\lambda}_1, ..., \bar{\lambda}_D\right)$ is the diagonal matrix of the signal subspace eigenvalues.

The R_x matrix is a Hermitian matrix and $R_x^H = R_x$, its eigenvalues are real numbers, and the eigenvectors are orthogonal. Assuming that AR_sA^H is full-rank, and $Rank(AR_sA^H) = D$, Λ_s contains the D high signal eigenvalues. The remaining $M-D$ eigenvalues are of noise character. Estimation of the signal arrival direction by *MUSIC* uses the orthogonality property of the U_n noise eigenvectors and of the A matrix:

$${U_n}^H a(\Theta) = 0, \Theta \in \{\Theta_1, \Theta_2, ..., \Theta_D\}.$$

Spatial spectrum:

$$P(\Theta) = \frac{1}{a^H(\Theta)U_n {U_n}^H a(\Theta)}. \qquad (7)$$

MUSIC method algorithm

1. Calculation of the R_x correlation matrix (which in practice is replaced by matrix (5)) and its expansion into eigenvalues and vectors.

2. Sorting eigenvalues in the descending order $\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq .... \geq \bar{\lambda}_M > 0$.

3. $M-D$ selection of the U_n, eigenvectors corresponding to the lowest eigenvalues.

4. Spectrum estimation according to (7).

In practice, the number of the D signal emitters is often unknown; thus, the task of selecting the required number of noise vectors is becoming more complicated. Some sources provide recommendations for such a task [10, 12].

***ESPRIT* invariant rotation method (*Estimation of Signal Parameters via Rotational Invariance Techniques*).** This method is a decomposition method for estimating frequencies. Unlike *MUSIC*, it does not require knowing and storing configuration of the antenna array [8]. In practice, modification of the basic algorithm is most often used based on the *TLS ESPRIT* generalized least squares method. *TLS ESPRIT* algorithm stages are presented below. Detailed description could be found in [13–15].

As in *MUSIC*, the $\hat{R}_x$ correlation matrix is estimated first, and its eigenvectors and its eigenvalues are calculated.

If necessary, number of the D signals is estimated, and eigenvectors corresponding to the highest eigenvalues are selected. From the selected eigenvectors, a basis matrix is generated that creates the $\overline{B} = [v_1, v_2, ..., v_D]$ signal subspace.

Next, the basis matrix is divided into B and B', blocks containing the first and the last $M-1$ rows of the $\overline{B}$ matrix, respectively:

$$\overline{B} = \begin{pmatrix} B \\ b_M \end{pmatrix} = \begin{pmatrix} b_0 \\ B' \end{pmatrix},$$

where b_0, and b_M are the first and the last rows of the generalized basis matrix.

The V matrix of the right singular vectors of the $\left[BB'\right]$ composite matrix is calculated and divided into four submatrices of the $D \times D$ size:

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{pmatrix}.$$

The $\overline{\lambda}_1, \overline{\lambda}_1, ..., \overline{\lambda}_D$ eigenvalues of the $\Psi_{TLS} = -V_{12}V_{22}{}^{-1}$ matrix are calculated. Desired angles are found as $\Theta_k = \arcsin\left(\arg(\overline{\lambda}_k)(\lambda / d)\right)$.

Comparison of the normalized spatial spectra obtained by estimation with the considered methods is presented in Fig. 2. Data was obtained for a TM model in configuration with eight receiving antenna elements. Emitter sources are two objects at the same distance of 50 m, moving at a speed of –5 and 5 m/s, and with angular position of –5 and 5°.

Periodogram method in Fig. 2, *a* was performed using the FFT with window transformation and with supplementing the initial sampling with zeros to the

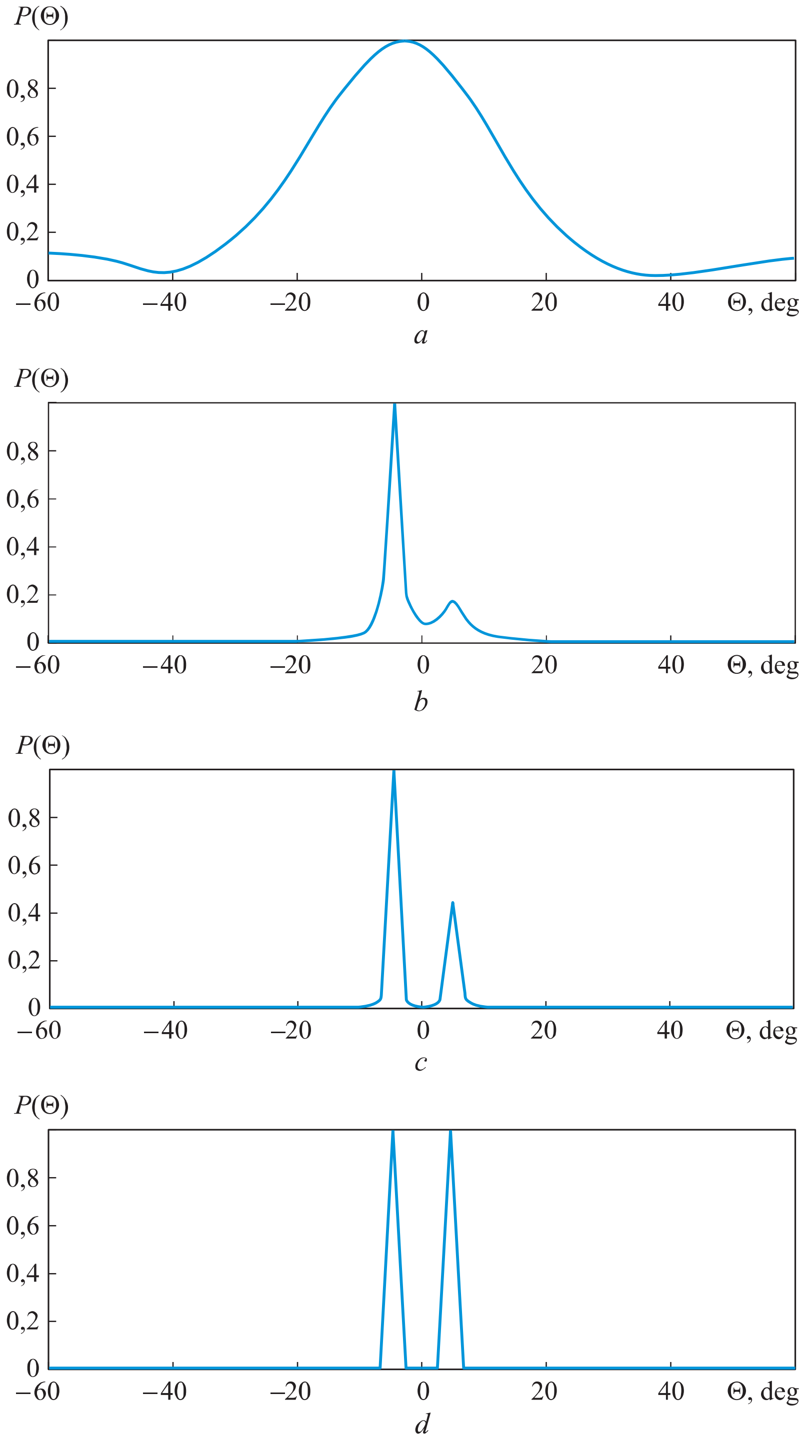

Fig. 2. Normalized spatial spectrum estimation by FFT (*a*), MD (*b*), *MUSIC* (*c*), *TLS ESPRIT* (*d*) methods

required size. It should be noted that *TLS ESPRIT* does not provide the signal power distribution depending on the angle; therefore, Fig. 2, *d* shows a pseudo spectrum constructed on the angular estimation.

Qualitative estimation of the described above methods operation in real sampling was based on the output data sets in the angle – distance coordinates. The angle – distance matrices for each method are presented in Fig. 3. Images are provided in pseudo colors, i.e., minimum value is in blue, maximum value is in yellow. Data were obtained from TM with a probing FMCW signal in the *MIMO* 2Tx/4Rx configuration.

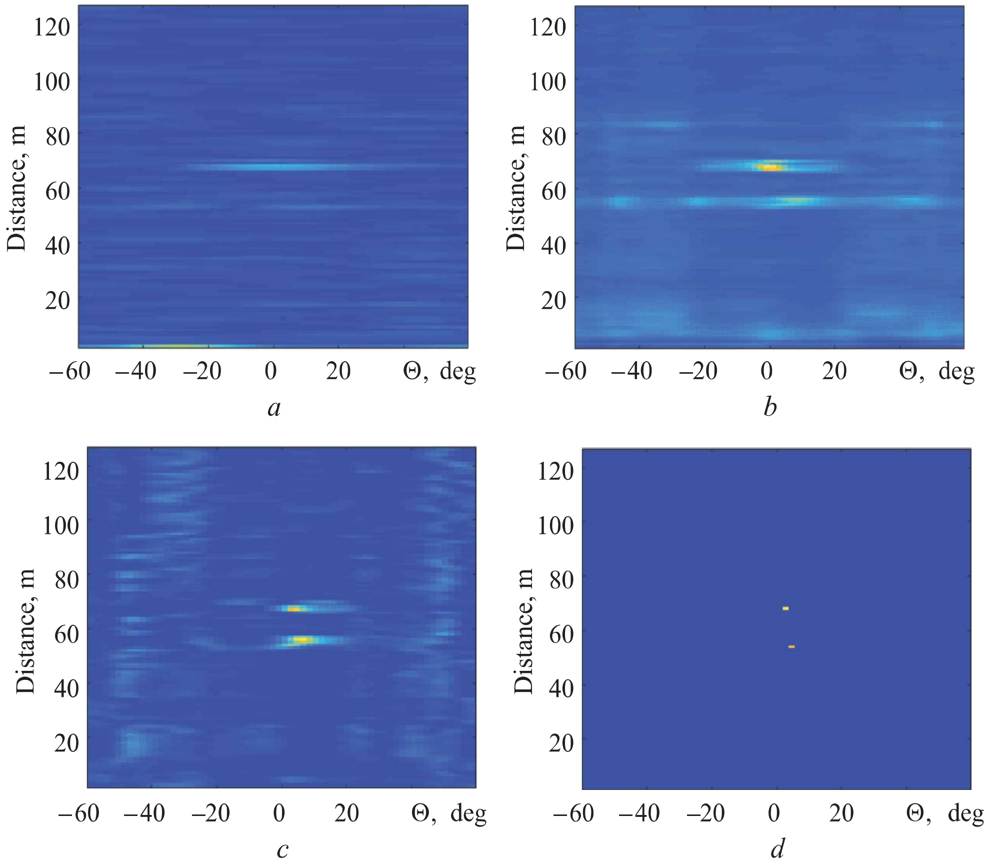

Fig. 3. Angle – distance matrices for FFT (*a*), MD (*b*), *MUSIC* (*c*) and *TLS ESPRIT* (*d*)

In the case under consideration, targets are in different distance channels. Estimation using *TLS ESPRIT* (Fig. 3, *d*) is provided by the distance indication of interest after targets detection. Matrix low contrast in Fig. 3, *a* indicates weak selectivity of the periodogram method. MD (Capon) and *MUSIC* methods present the similar results.

Conclusion. Results obtained provide a visual representation of the methods' ability to resolve objects at their different positions in range, speed and azimuth. Periodogram method is potentially unable to distinguish objects that are adjacent in the angle. MD, *MUSIC* and *TLS ESPRIT* methods are more acceptable in tasks of signal emitters localization. Decomposition methods based on *MUSIC* and *ESPRIT* ensure higher resolution than MD, but also require significant computational expenses. *TLS ESPRIT* does not allow constructing signal power distribution in the angle – distance coordinates, which could become critical in certain tasks. However, it uses to a major extent the deterministic relationship between orthogonal subspaces compared to *MUSIC* and does not require knowing and storing the antenna array configuration. Selection of this or that method depends on functional tasks, system parameters, computational resources and other criteria required in a radar sensor.

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