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SOLVING THE PROBLEM OF THE OPTIMAL CONTROL SYSTEM GENERAL SYNTHESIS BASED ON APPROXIMATION OF A SET OF EXTREMALS USING THE SYMBOL REGRESSION METHOD

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Abstract

A new approach is considered to solving the problem of synthesizing an optimal control system based on the extremals' set approximation. At the first stage, the optimal control problem for various initial states out of a given domain is being numerically sold. Evolutionary algorithms are used to solve the optimal control problem numerically. At the second stage, the problem of approximating the found set of extremals by the method of symbolic regression is solved. Approach considered in the work makes it possible to eliminate the main drawback of the known approach to solving the control synthesis problem using the symbolic regression method, which consists in the fact that the genetic algorithm used in solving the synthesis problem does not provide information about proximity of the found solution to the optimal one. Here, control function is built on the basis of a set of extremals; therefore, any particular solution should be close to the optimal trajectory. Computational experiment is presented for solving the applied problem of synthesizing the four-wheel robot optimal control system in the presence of phase constraints. It is experimentally demonstrated that the synthesized control function makes it possible for any initial state from a given domain to obtain trajectories close to optimal in the quality functional. Initial states were considered during the experiment, both included in the approximating set of optimal trajectories and others from the same given domain. Approximation of the extremals set was carried out by the network operator method

Keywords

Optimal control, control synthesis, extremals, evolutionary algorithms, symbolic regression method, network operator method

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Introduction. Task of the optimal control general synthesis is in finding a control function, which arguments include components of the control object state vector. After substituting the found control function in the control object model right part, a system of ordinary differential equations without free control vector is obtained. Such a model is often called the closed-loop control model. According to the problem statement of ensuring the optimal control system general synthesis, particular solution of this system from any initial state of a certain given state space area appears to be optimal in regard to a certain given criterium in the trajectory quality.

The fact that initial conditions belong to the entire state space or its certain domain along with the requirement to find optimal control in the form of a multidimensional function from the state space coordinates appears to be the main complexity and the main difference between the synthesis problem and the optimal control problem, which requires to find control in the form of a time function for certain initial conditions.

Despite the importance of the optimal control synthesis problem, exact methods for solving it are currently missing. Analytical solutions to the synthesis problem are known only in regard to simple objects of small dimension. It is not possible to obtain an analytical solution for major part of applied control objects.

The optimal control general synthesis problem was so named and formulated by V.G. Boltyansky in the late 1960s [1] immediately after the optimal control problem formulation [2]. V.G. Boltyansky and L.S. Pontryagin solved several problems of general control synthesis based on the Pontryagin maximum principle. Solution appeared to be in switching condition on a set of extremals, movement along which to the terminal state provided obtaining the functional optimal value.

Bellman equation is often used to solve the synthesis problem, and it is a system of equations in partial derivatives. Bellman equation analytical solutions are known only for not complicated systems. Dynamic programming method developed by R. Bellman is mostly often used in numerical solution. As a result of applying this method, control function analytical expression could not be obtained, but there appears a set control vector values for the set of state vector values. All known solutions to the dynamic programming method control synthesis problem are using only one initial condition due to the "dimension curse"; therefore, this numerical method is not used to solve the problem of optimal control general synthesis.

Analytical designing of optimal regulator (ADOR) method is the most well-known technique in solving the synthesis control problem [3]. It was designed for object control linear models and quadratic quality criterion, and makes it possible to find matrix elements of the state vector linear function by solving the Riccati equations system.

Known methods in solving the synthesis problem based on using the Lyapunov function [4], analytical designing of aggregated regulator (ADAR) [5] and other methods are not universal, and were successfully used only in certain models of a control object [6, 7].

Universal numerical methods appeared in the beginning of the XXI century to solve the optimal control general synthesis problem based on symbolic regression methods [8]. All these methods find mathematical expressions for a synthesizing function in encoded form using the evolutionary algorithms. The difference between the symbolic regression methods lies in the mathematical expression coding technique and in the search algorithms. Main disadvantage of this approach is that the evolutionary algorithm does not make it possible to determine, how the solution found is close to the optimal one.

The present paper proposes to use an approach aimed at eliminating the indicated drawback. A set of optimal trajectories is initially constructed, i.e., the optimal control problem for all initial conditions from a given bounded set of state space is repeatedly solved. At the second stage, approximation of the found set of optimal trajectories is performed using the network operator method, which belongs to the symbolic regression methods class [9]. This approach makes it possible to solve the optimal control synthesis problem and to find a synthesizing function that provides particular solution to the closed-loop system model from any initial state out of a limited set optimal according to the given quality criterion.

Approach considered in the work stays within the framework of modern trends in constructing control systems based on the learning methods. Here, building a set of optimal trajectories corresponds to creation of a learning sample, and approximation of the resulting set corresponds to the learning structure of symbolic regression method. At the same time, quality of learning is evaluated by solutions obtained using the symbolic regression method for initial conditions that are not considered in the learning set of optimal trajectories.

Optimal control general synthesis problem. Let us present formulation of the optimal control synthesis problem. Let mathematical model of the control object be given in the following form:

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{u}\right), \tag{1}$$

where $\mathbf{x} = \left[x_1, \ldots, x_n\right]^t$ is the object state vector, $\mathbf{x} \in \mathbb{R}^n$; $\mathbf{u} = \left[u_1, \ldots, u_m\right]^t$ is the object control vector; $\mathbf{u} \in \mathrm{U} \subseteq \mathbb{R}^m$, U is bounded closed set.

Set of initial states

$$\mathrm{X}_0 \subseteq \mathbb{R}^n \tag{2}$$

and terminal conditions

$$\mathbf{x}\left(t_f\right) = \mathbf{x}^f, \tag{3}$$

where t_f is the control process limited time, which could be set or determined by achieving the terminal conditions.

Quality functional has the following form:

$$J = \int_0^{t_f} f_0\left(\mathbf{x}(t), \mathbf{u}(t)\right) dt \to \min. \tag{4}$$

It is required to find control in the form of a multidimensional function of the object state space vector components:

$$\mathbf{u} = \mathbf{h}(\mathbf{x}), \tag{5}$$

where $\mathbf{h}(\mathbf{x}): \mathbb{R}^n \to \mathbb{R}^m$, $\mathbf{h}(\mathbf{x}) \subseteq \mathrm{U}$, $\forall \mathbf{x} \in \mathbb{R}^n$.

Let us write down the control object mathematical model (1) in the form of a following system:

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{h}(\mathbf{x})\right), \tag{6}$$

which solution for any object initial state in the $\forall \mathbf{x}(0) = \mathbf{x}_0 \in \mathrm{X}_0$ given region is the $\mathbf{x}(t, \mathbf{x}_0)$ time vector function. This function ensures the control object displacement from the $\mathbf{x}(0)$ initial state to the $\mathbf{x}^f$ terminal position in the $t_f < \infty$ finite time and at the same time delivers minimum to the quality function (4):

$$\min_{\mathbf{u} \in \tilde{\mathrm{U}}} \int_0^{t_f} f_0\left(\mathbf{x}(t, \mathbf{x}_0), \mathbf{u}(t)\right) dt, \tag{7}$$

where $\tilde{\mathrm{U}} = \left\{\tilde{\mathbf{u}}(\cdot)\right\}$ is the set of all acceptable controls satisfying the $\tilde{\mathbf{u}}(t) \subseteq \mathrm{U}$, $0 \le t \le t_f$ constraints and ensuring achievement of the terminal conditions (3).

Solution to the problem determined of synthesizing the optimal control requires searching for a multidimensional function (5) that satisfies optimality condition (7) for all possible initial values from the set (2). Analytical solution of the problem determined is possible only for not complicated objects of small dimension, which models in the form (6) have analytical solutions. For most applied control objects, it is not possible to receive an analytical solution.

For system (1), it is possible to obtain a particular numerical solution for a single $\mathbf{x}(0)$ initial state out of the set (2) optimal by the functional value (4). Such a solution would coincide for a given initial state with the solution that could be obtained using the $\mathbf{h}(\mathbf{x})$ desired function. However, it is not possible to determine the structure of multidimensional control function from one particular solution, since the found particular solution may not provide optimal control from another initial state from the set (2). According to formulation of the optimal control synthesis problem, multidimensional control function should ensure optimal solution for all initial states from the set (2).

The paper considers application of the numerical method in solving the optimal control synthesis problem for finding a multidimensional function (5). Let us formulate a problem statement for the optimal control numerical synthesis. Let us replace the continuous set (2) with a finite set of *N* elements:

$$\tilde{\mathrm{X}}_0 = \left(\mathbf{x}_0^1, \ldots, \mathbf{x}_0^N\right). \tag{8}$$

For each initial state from the set (8), particular numerical solution of system (6) should provide an optimal value of the functional (4).

Solving the problem of optimal control numerical synthesis based on the optimal trajectories' approximation. At the first stage, the optimal control problem (1), (3), (4) is solved for each initial state from the set (8). Next, the search for the multidimensional function (5) is conducted by approximating the found set of optimal trajectories. To do this, the following methods could be used: genetic programming [10], grammatical evolution [11], analytical programming [12], or other methods belonging to the symbolic regression methods class, where using evolutionary algorithms the code for the optimal mathematical expression is searched. The optimal trajectories' approximation problem could be solved using the neural networks popular these days. Neural network learning is tuning or searching for a large number of parameters, rather than searching for a mathematical expression. Thus, it is not possible to obtain the structure of a desired function in explicit form using a neural network.

Statement of the optimal trajectories' approximation problem is presented. Suppose that for all initial conditions (8) there are solutions to the optimal control problem of a mathematical model (1) that satisfy quality functional (4) and terminal conditions (3). Then these solutions are a set of pairs of optimal trajectories and program controls:

$$\tilde{\mathrm{D}} = \left\{\left(\tilde{\mathbf{x}}^1(\cdot), \tilde{\mathbf{u}}^1(\cdot)\right), \ldots, \left(\tilde{\mathbf{x}}^N(\cdot), \tilde{\mathbf{u}}^N(\cdot)\right)\right\}, \tag{9}$$

where $\tilde{\mathbf{x}}^i(\cdot)$ is particular solution to the $\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \tilde{\mathbf{u}}^i(t)\right)$ system of equations with the $\mathbf{x}(0) = \mathbf{x}_0^i$ initial conditions; $\tilde{\mathbf{u}}^i(\cdot)$ is the solution to the optimal control problem for a given initial condition and taking into account the $\tilde{\mathbf{u}}^i(t) \in \mathrm{U} \subseteq \mathbb{R}^m$, $i = \overline{1, N}$ constraints.

To numerically approximate the set (9), let us introduce time discretization. Let us set the $\Delta t > 0$ small value and determine a set of discrete time values for each solution from the set (8):

$$\mathrm{T}_i = \left(0, \Delta t, 2\Delta t, \ldots, M_i \Delta t\right),$$

where $M_i = \left\lceil \frac{t_f}{\Delta t} \right\rceil$, $t_f = \max\left\{t_{1,f}, \ldots, t_{N,f}\right\}$, $t_{i,f}$, is the time of control process for the $\mathbf{x}_0^i$ initial state, $i = \overline{1, N}$. Then, state and control vectors' values for each initial state at the $t_i \in \mathrm{T}_i$ discrete time instant could be written down as:

$$\tilde{\mathbf{x}}^{i,j} = \tilde{\mathbf{x}}^i\left(t_{i,j}\right);$$

$$\tilde{\mathbf{u}}^{i,j} = \tilde{\mathbf{u}}^i\left(t_{i,j}\right),$$

where $j = \overline{1, M_i}$, $i = \overline{1, N}$.

As a result, a set of points of optimal trajectories and program controls is obtained in the $\mathbb{R}^n \times \mathbb{R}^m$:

$$\mathrm{D} = \\ = \left\{\left(\left(\tilde{\mathbf{x}}^{1,1}, \tilde{\mathbf{u}}^{1,1}\right), \ldots, \left(\tilde{\mathbf{x}}^{1,M_1}, \tilde{\mathbf{u}}^{1,M_1}\right)\right), \ldots, \left(\left(\tilde{\mathbf{x}}^{N,1}, \tilde{\mathbf{u}}^{N,1}\right), \ldots, \left(\tilde{\mathbf{x}}^{N,M_N}, \tilde{\mathbf{u}}^{N,M_N}\right)\right)\right\}. \tag{10}$$

Based on approximation of the set points (10), let us synthesize a multidimensional control function (5) that satisfies the criterion:

$$J = \left\| \sum_{i=1}^{N} \sum_{j=1}^{M_i} \left(\tilde{\mathbf{u}}^{i,j} - h\left(\tilde{\mathbf{x}}^{i,j}\right)\right) \right\| \to \min.$$

This work proposes to implement the network operator method, which belongs to the symbol regression methods class, to approximate optimal trajectories and search for the synthesizing control function [9, 13]. Network operator method uses representation of the desired mathematical expression in the form of a directed graph, and in the computer memory it is used as an integral matrix. Each graph node-source being an element on the matrix main diagonal is connected to one of the synthesizing function argument. The remaining nodes are connected to binary operations. The graph arcs, which in the matrix representation are elements above the main diagonal, are connected to the unary operations. The set of operations in the network operator is limited. Search for the network operator optimal structure is carried out by a genetic algorithm on a set of the basic solution options, which, as a rule, is set on the basis of the specific problem analytical study. Selecting a proper basic solution could produce a positive effect on the search rate, but in general it does not affect quality of the solution obtained. Unlike genetic programming, network operator method does not require lexical analysis of a character string, but makes it possible to quickly calculate the value of a function from the matrix strings. Advantage of the network operator over neural networks lies in the ability to obtain structure of the desired synthesizing function in the explicit form. Optimal control calculated by such function after substitution in model (1) provides a particular solution from any initial state from set (8), which is optimal according to the given quality criterion.

Computational experiment. As a computational experiment, the problem of synthesizing optimal control over the four-wheeled mobile robot displacement from a limited space region to the terminal position was being solved.

Mathematical model of a four-wheeled mobile robot has the following form [14]:

$$\begin{aligned} \dot{x}_1 &= u_1 \cos x_3; \\ \dot{x}_2 &= u_1 \sin x_3; \\ \dot{x}_3 &= \frac{u_1}{L} \tan u_2, \end{aligned} \tag{11}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^t$ is the object state vector; $\mathbf{u} = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^t$ is the object control vector; and L is the wheelbase.

Constraints are set for the control vector:

$$\mathbf{u}^- \le \mathbf{u} \le \mathbf{u}^+,$$

where $\mathbf{u}^- = \begin{bmatrix} u_1^- & u_2^- \end{bmatrix}^t$ and $\mathbf{u}^+ = \begin{bmatrix} u_1^+ & u_2^+ \end{bmatrix}^t$, and for the control process duration: $t_{\max}$, $t_{\max} > 0$.

The set of initial states and terminal conditions is known

$$\mathrm{X}_0 = \left\{ \mathbf{x}^1(0), \mathbf{x}^2(0), \ldots, \mathbf{x}^N(0) \right\};$$

$$\mathbf{x}\left(t_{i,f}\right) = \mathbf{x}^f,$$

where $\mathbf{x}^i(0) = \mathbf{x}^{i,0} = \begin{bmatrix} x_1^{i,0} & x_2^{i,0} & x_3^{i,0} \end{bmatrix}^t$; $\mathbf{x}(t_{i,f}) = \mathbf{x}^f = \begin{bmatrix} x_1^f & x_2^f & x_3^f \end{bmatrix}^t$, $t_{i,f}$ is the control process time, $0 < t_{i,f} \le t_{\max}$, $t_{i,f}$ is not specified, but is determined from the relation:

$$t_{i,f} = \begin{cases} t, \text{ if } t < t_{\max} \text{ and } \left\| \mathbf{x}^i(t) - \mathbf{x}^f \right\| \le \varepsilon; \\ t_{\max} \text{ otherwise}, \ i = \overline{1, N}, \end{cases}$$

ε is the given minor positive value; N is the size of the set of initial states.

Phase constraints were set

$$h_j(\mathbf{x}) = r_j^* - \sqrt{\left(x_{j,1}^* - x_1\right)^2 + \left(x_{j,2}^* - x_2\right)^2} \le 0,$$

where r_j^*, $x_{j,1}^*$ and $x_{j,2}^*$ are the phase constraint specified parameters, $j = \overline{1, K}$, K is the number of phase constraints.

At the first stage, the problem of finding extremals was being solved. For each $\mathbf{x}^i(0) \in \mathrm{X}_0$, $i = \overline{1, N}$ initial state, a search was made for control vectors as the $\mathbf{u}^i(t)$ time functions that move the mobile robot from the $\mathbf{x}^{i,0}$ given initial position to the $\mathbf{x}^f$ terminal position in a minimum time period. For each search, the minimized quality functional was

$$J_i = t_{i,f} + \left\| \mathbf{x}^i(t_{i,f}) - \mathbf{x}^f \right\| + \int_0^{t_{i,f}} \left(\sum_{j=1}^{K} \alpha_j \vartheta\left(h_j(\mathbf{x})\right) h_j(\mathbf{x}) \right) dt \to \min,$$

where $i = \overline{1, N}$;

$$\vartheta\left(h_j(\mathbf{x})\right) = \begin{cases} 1, \text{ if } h_j(\mathbf{x}) > 0; \\ 0 \text{ otherwise} \end{cases}$$

is a Heaviside function; α_j are the given penalty factors, $j = \overline{1, K}$.

Solution search was carried out by reducing the initial problem of discontinuous optimization to the nonlinear programming problem using the piecewise linear approximation. To do this, a minor $\Delta t > 0$ interval is set, and the number of intervals is determined:

$$M = \left[\frac{t_{\max}}{\Delta t} \right].$$

The $\tilde{\mathbf{u}}^i(t) = \begin{bmatrix} \tilde{u}_1^i(t) & \tilde{u}_2^i(t) \end{bmatrix}^t$ control value at the t time is determined from the relation:

$$\tilde{u}_1^i(t) = \begin{cases} u_1^-, \text{ if } q_{i,j} + (q_{i,j+1} - q_{i,j})\dfrac{(t-(j-1)\Delta t)}{\Delta t} < u_1^-; \\ u_1^+, \text{ if } q_{i,j} + (q_{i,j+1} - q_{i,j})\dfrac{(t-(j-1)\Delta t)}{\Delta t} > u_1^+; \\ q_{i,j} + (q_{i,j+1} - q_{i,j})\dfrac{(t-(j-1)\Delta t)}{\Delta t} \text{ otherwise;} \end{cases}$$

$$\tilde{u}_2^i(t) = \begin{cases} u_2^-, \text{ if } q_{i,k} + (q_{i,k+1} - q_{i,k})\dfrac{(t-(j-1)\Delta t)}{\Delta t} < u_2^-; \\ u_2^+, \text{ if } q_{i,k} + (q_{i,k+1} - q_{i,k})\dfrac{(t-(j-1)\Delta t)}{\Delta t} > u_2^+; \\ q_{i,k} + (q_{i,k+1} - q_{i,k})\dfrac{(t-(j-1)\Delta t)}{\Delta t} \text{ otherwise,} \end{cases}$$

where $i = \overline{1, N}$, $j\Delta t \le t < (j+1)\Delta t$, $j = \overline{1, M}$, $k = \overline{M+1,\ 2M}$.

Thus, solution to this problem for each $\mathbf{x}^{i,0}$ object initial state is the $\mathbf{q}^i = \left[q_{i,1}, \ldots, q_{i,p}\right]^t$ constant parameters vector, $\mathbf{q}^i \in \mathbb{R}^p$, $i = \overline{1, N}$, $q_j^- \le q_j \le q_j^+$, q_j^-, q_j^+ are the specified values of parameter constraints, $j = \overline{1, p}$, $p = 2(M+1)$. Search for the $\mathbf{q}^i$ parameter vectors is carried out by one of the unconditional optimization methods. Then, integration of differential equations (11) is carried out and extremals are built on the basis of the obtained solution.

Model parameters in the computational experiment had the following values: $u_1^- = -10$, $u_1^+ = 10$, $u_2^- = -1$, $u_2^+ = 1$, $t_{\max} = 2.5$, $\varepsilon = 0.01$, $\Delta t = 0.25$; number of intervals $M = \lceil t_{\max} / \Delta t \rceil = 10$; parameter vector dimension $\mathbf{q}^i$ $p = 2(M+1) = 22$; number of phase constraints $K = 2$, $x_{1,1}^* = 1.5$, $x_{1,2}^* = 5$, $r_1^* = 3$, $x_{2,1}^* = 8.5$, $x_{2,2}^* = 5$, $r_2^* = 3$, $\alpha_j = 5$, $j = \overline{1, K}$, $x_1^f = 0$, $x_2^f = 0$, $x_3^f = \pi$.

The set of initial conditions was constrained

$$\begin{aligned} 7 \le x_{0,i,1} \le 11; \\ 9 \le x_{0,i,2} \le 11; \\ 0 \le x_{0,i,3} < 2\pi, \end{aligned}$$

$i = \overline{1, N}$, $N = 7$.

Bee algorithm [15, 16], one of the most efficient algorithms in solving this class of problems [17, 18], was selected as the method of unconditional optimization for solving the obtained nonlinear programming problem. Parameters of the algorithm had the following values: size of the set of possible solutions $H = 32$, number of the algorithm iterations $W = 30\,000$, algorithm coefficients $\alpha = 0.7278$, $\beta = 0.5$, $\gamma = 0.1$, $\delta = 1$.

As a result of calculations for each initial condition from the X_0 set, the optimal control was obtained, on which basis the extremals were built. Values of the minimized quality functional for each initial condition are presented in Table 1. Fig. 1 shows optimal trajectories of the mobile robot displacement from the X_0 initial states plurality to the $\mathbf{x}^f$ terminal position.

Table 1

Quality functional values for the extremals found

$x_{i,1}^0$	$x_{i,2}^0$	$x_{i,3}^0$	J_i
7	10	π	1.60
8	9	π	1.41
8	11	π	1.51
9	10	π	1.49
10	9	π	1.47
10	11	π	1.57
11	10	π	1.85

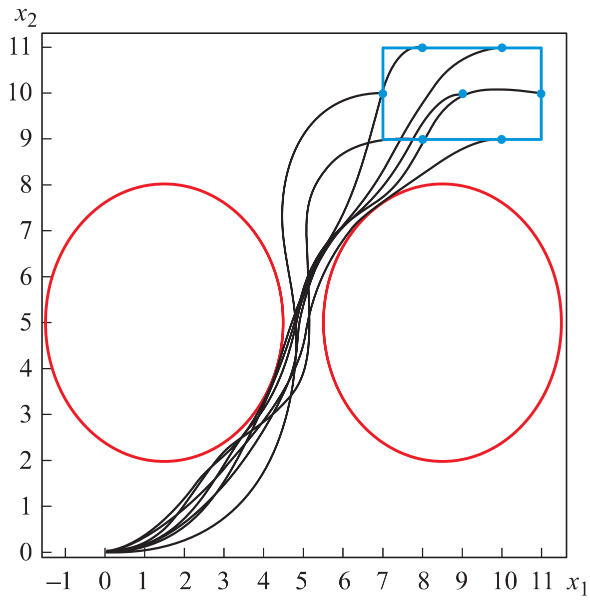

Fig. 1. Mobile robot displacement optimal trajectories from different initial conditions

At the second stage of the computational experiment, obtained extremals were used to synthesize optimal control using the network operator method. Parameters of the network operator method had the following values: network operator matrix size 40; number of possible solutions in the initial population 256; number of generations 25 000. Structure and solution parameters were searched using the variational genetic algorithm with a multiple basis [19]. Number of possible crossed pairs in a generation was 128; number of variations in one solution 8; probability of mutation 0.7; number of bases 128; number of elite decisions 8; number of generations between changing the bases 16.

Simultaneously with searching for synthesized optimal control optimal structure, search was made for the parameter vector optimal value. Each possible solution was a network operator matrix and a parameter vector.

As a result of computational experiment, solution was obtained in the form of a network operator matrix, which corresponded to the following expression:

$$\mathbf{h}(\mathbf{x}) = \chi_1\left(z_{24}^{-1},\ \ln(|z_{23}|),\ z_{17},\ z_{13}^{-1},\ z_{10}^{-1},\ e^{z_9},\ z_8^{-1},\ \tanh(0.5q_7)\right);$$

$$z_{24} = \sqrt[3]{z_{21}}z_{18}^{-1}z_8^{-1}\left(z_3 - z_3^3\right)\arctan(q_{11})q_9^{-1}(-q_7)x_1;$$

$$z_{23} = \max\Big(\ln(|z_{22}|),\ e^{z_{21}},\ \tanh(0.5z_{20}),\ z_{19}^3,\ z_{17}^{-1},\ \mathrm{sgn}(z_{15})\sqrt{|z_{15}|}, \Rightarrow$$
$$\Rightarrow -z_{13},\ \tanh(0.5z_8),\ z_6,\ z_5,\ e^{z_4},\ q_8 - q_8^3,\ x_3\Big);$$

$$z_{22} = -z_{21}\,\mathrm{sgn}(z_{19})\sqrt{|z_{19}|}z_{16}^2\, z_{14}\, z_{13}\,\mathrm{sgn}(z_{10})\sqrt{|z_{10}|}z_7\, z_3^2\ \times$$
$$\times \arctan(q_{11})q_{10}\, q_6\left(q_1 - q_1^3\right)x_3^2\sqrt[3]{x_2}\tanh(0.5x_1);$$

$$z_{21} =$$
$$= \max\begin{pmatrix} \mathrm{sgn}(z_{19})\sqrt{|z_{19}|},\ z_{18}^3,\ -z_{17},\ \sqrt[3]{z_{16}},\ z_{14}^3,\ z_{13}^3, \Rightarrow \\ \Rightarrow \ln(|z_{12}|),\ z_{10}^3,\ \arctan(z_8),\ \mathrm{sgn}(z_7)\sqrt{|z_7|},\ -z_6, \Rightarrow \\ \Rightarrow \ln(|z_5|),\ \sqrt[3]{z_4},\ z_3,\ q_{11}^3,\ q_{10} - q_{10}^3,\ q_9 - q_9^3,\ \ln(|q_8|), \Rightarrow \\ \Rightarrow \mathrm{sgn}(q_7)\sqrt{|q_7|},\ \arctan(q_6),\ q_1 \end{pmatrix};$$

$$z_{20} =$$
$$= \chi_2\begin{pmatrix} \sqrt[3]{z_{18}},\ \arctan(z_{17}),\ e^{z_{16}},\ e^{z_{14}},\ e^{z_{13}},\ \mathrm{sgn}(z_{11})\sqrt{|z_{11}|}, \Rightarrow \\ \Rightarrow \sqrt[3]{z_8}, \mathrm{sgn}(z_7)\sqrt{|z_7|},\ -z_4, \Rightarrow \\ \Rightarrow \arctan(q_{12}),\ \arctan(q_6),\ q_3^2, \ln(|q_1|),\ \arctan(x_3), \Rightarrow \\ \Rightarrow \tanh(0.5x_2),\ \tanh(0.5x_1) \end{pmatrix};$$

$$z_{19} = z_{18}^3\ln(|z_{17}|)\tanh(0.5z_{16})\left(z_{13} - z_{13}^3\right)\sqrt[3]{z_{10}}e^{z_8}\arctan(z_7)\ln(|z_5|)e^{z_3}\ \times$$
$$\times \arctan(z_1)\arctan(q_9)\left(q_8 - q_8^3\right)e^{q_7}\, q_4^2\,\mathrm{sgn}(q_2)\sqrt{|q_2|}\sqrt[3]{x_2};$$

$$z_{18}=\arctan(z_{15})+e^{z_{14}}+\operatorname{sgn}(z_{13})\sqrt{|z_{13}|}+\ln(|z_{11}|)+\operatorname{sgn}(z_{10})\sqrt{|z_{10}|}+$$
$$+e^{z_8}+\ln(|z_6|)+\operatorname{sgn}(z_5)\sqrt{|z_5|}+z_1+\sqrt[3]{q_{12}}+q_8+$$
$$+\ln(|q_6|)+q_5^2+q_4^{-1}+q_3^3-x_3+x_2;$$

$$z_{17}=z_{16}+e^{z_{13}}+z_7^{-1}+z_1^{-1}+\sqrt[3]{q_{10}}+\arctan(x_3);$$

$$z_{16}=\min\Big(\arctan(z_{14}),\ z_{11}^3, \operatorname{sgn}(z_9)\sqrt{|z_9|},\ \arctan(z_8),\Rightarrow$$
$$\Rightarrow\tanh(0.5q_{11}),\ -q_{10},\ q_7,\ -q_6,\ \sqrt[3]{q_5}\Big);$$

$$z_{15}=\chi_2\Big(z_{14}^3,\ z_{11}^2,\ \arctan(z_{10}),\ z_5-z_5^3,\ z_4^2,\ z_2^3,\ \tanh(0.5q_8),\Rightarrow$$
$$\Rightarrow q_6^{-1},\ e^{q_5},\ q_4^{-1},\ \arctan(q_3),\ \arctan(x_3),\ \sqrt[3]{x_1}\Big);$$

$$z_{14}=\left(z_{13}-z_{13}^3\right)z_{10}^3\ln(|z_7|)\,z_5^3\ln(|z_3|)\left(z_2-z_2^3\right)\sqrt[3]{z_1}\times$$
$$\times\tanh(0.5q_{10})\tanh(0.5q_9)\,q_8^2\sqrt[3]{q_5}q_3^3e^{q_2};$$

$$z_{13}=z_{11}+z_{10}+\ln(|z_8|)+\tanh(0.5z_6)+\ln(|z_4|)+$$
$$+\tanh(0.5z_2)+e^{q_7}+\sqrt{q_4}+q_3+q_2+x_2^{-1}+x_1^{-1};$$

$$z_{12}=\chi_1\Big(z_8-z_8^3,\ \operatorname{sgn}(z_6)\sqrt{|z_6|},\ q_{12},\ q_{11},\ q_3^3,\ x_3\Big);$$

$$z_{11}=\chi_1\Big(e^{z_9},\ z_8,\ e^{z_6},\ e^{z_3},\ q_{11}\Big);$$

$$z_{10}=z_7+z_6^2+\sqrt[3]{z_3}+\operatorname{sgn}(z_2)\sqrt{|z_2|}+\sqrt{q_{11}}-q_1+e^{x_3};$$

$$z_9=\chi_1\begin{pmatrix}\arctan(z_6),\ \operatorname{sgn}(z_5)\sqrt{|z_5|},\ \operatorname{sgn}(z_3)\sqrt{|z_3|},\ z_2^2,\ q_{12}^3,\ q_{11}^3,\ \ln(|q_{10}|),\Rightarrow\\ \Rightarrow q_7,\ \sqrt{q_6},\ \ln(q_4),\ \ln(q_3),\ \tanh(0.5q_1),\ x_3^3,\ \tanh(0.5x_1)\end{pmatrix};$$

$$z_8=z_5\tanh(0.5z_1)\,q_8;$$

$$z_7=z_4\arctan(q_{11})\,q_7\tanh(0.5q_2)\tanh(0.5x_1);$$

$$z_6=z_3+\ln(q_8)+q_6+q_5;$$

$$z_5=z_2+\sqrt[3]{z_1}+\sqrt{q_{11}}+q_5;$$

$$z_4 = \chi_2\left(\sqrt[3]{z_3},\ \ln\left(\left|z_2\right|\right),\ z_1,\ -q_8,\ q_5^{-1},\ q_4,\ \tanh\left(0.5x_2\right),\ x_1 - x_1^3\right);$$

$$z_3 = -q_3x_3x_1;\ z_2 = q_2x_2;\ z_1 = \tanh\left(0.5q_{10}\right)q_1\tanh\left(0.5x_3\right)x_1.$$

In the obtained expression $\chi_1\left(a_1, a_2\right) = a_1 + a_2 - a_1a_2$ and $\chi_2\left(a_1, a_2\right) =$ $= \operatorname{sgn}\left(a_1 + a_2\right)\sqrt{a_1^2 + a_2^2}$ are the commutative binary operations, parameters vector:

$$\mathbf{q} = \begin{bmatrix} 15.2290;\ 9.4905;\ 4.1550;\ 3.8315;\ 0.3091;\ 3.5425;\ \Rightarrow \\ \Rightarrow\ 5.4607;\ 7.0593;\ 8.9385;\ 11.5659;\ 15.2224;\ 2.0520 \end{bmatrix}^t.$$

To check the solution, let us obtain the optimal control for various initial states using the synthesizing function found. Let us consider among the initial states the X_0 both those present in the set and those not present. For each initial state, we also obtain solution to the optimal control problem and compare the quality functional value with the result received using the synthesizing function. Table 2 shows the quality functional values obtained using the synthesizing function (J_i^*) and by solving the optimal control problem (J_i) for various initial states. Fig. 2 presents graphs of the optimal trajectories for two initial conditions. Trajectory obtained using the synthesizing function is shown in gray and the one obtained by solving the optimal control problem — in black. Results received indicate high quality of the synthesizing function. For the trajectories obtained using the synthesizing function, deviation from the quality functional reference values received by solving the optimal control problem was: 0.01 (minimum); 0.07 (maximum); 0.0329 (average); 0.0278 (rms).

Table 2

Comparative computational experiment results

$x_{i,1}^0$	$x_{i,2}^0$	$x_{i,3}^0$	J_i^*	J_i
8	9	π	1.42	1.41
8	10	π	1.55	1.53
9	9	π	1.41	1.36
9	10	π	1.52	1.49
9	11	π	1.60	1.57
10	9	π	1.54	1.47
10	10	π	1.57	1.55

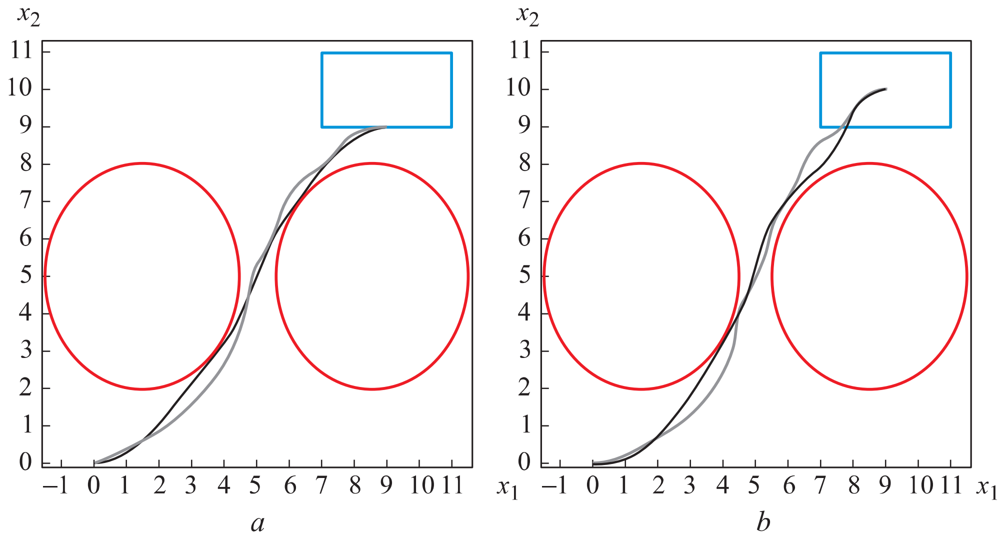

Fig. 2. Optimal trajectories obtained by solving the optimal control problem (black) and using the synthesizing function (gray) for the initial states: $x(0) = [9 \;\; 9 \;\; \pi]^t$ (*a*); $x(0) = [9 \;\; 10 \;\; \pi]^t$ (*b*)

Conclusion. As a result of using the network operator method for approximating the extremals, a synthesizing function was obtained that ensures the control object displacement from a limited domain with initial conditions to the given terminal state. Since the synthesizing function structure was searched on the basis of a set of optimal trajectories, it could be asserted that solutions obtained with its involvement are also optimal. Computational experiment results demonstrated insignificant deviation in the mobile robot motion trajectory obtained using the synthesizing function from the optimal one. This confirms high efficiency of the proposed optimal control synthesis method.

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